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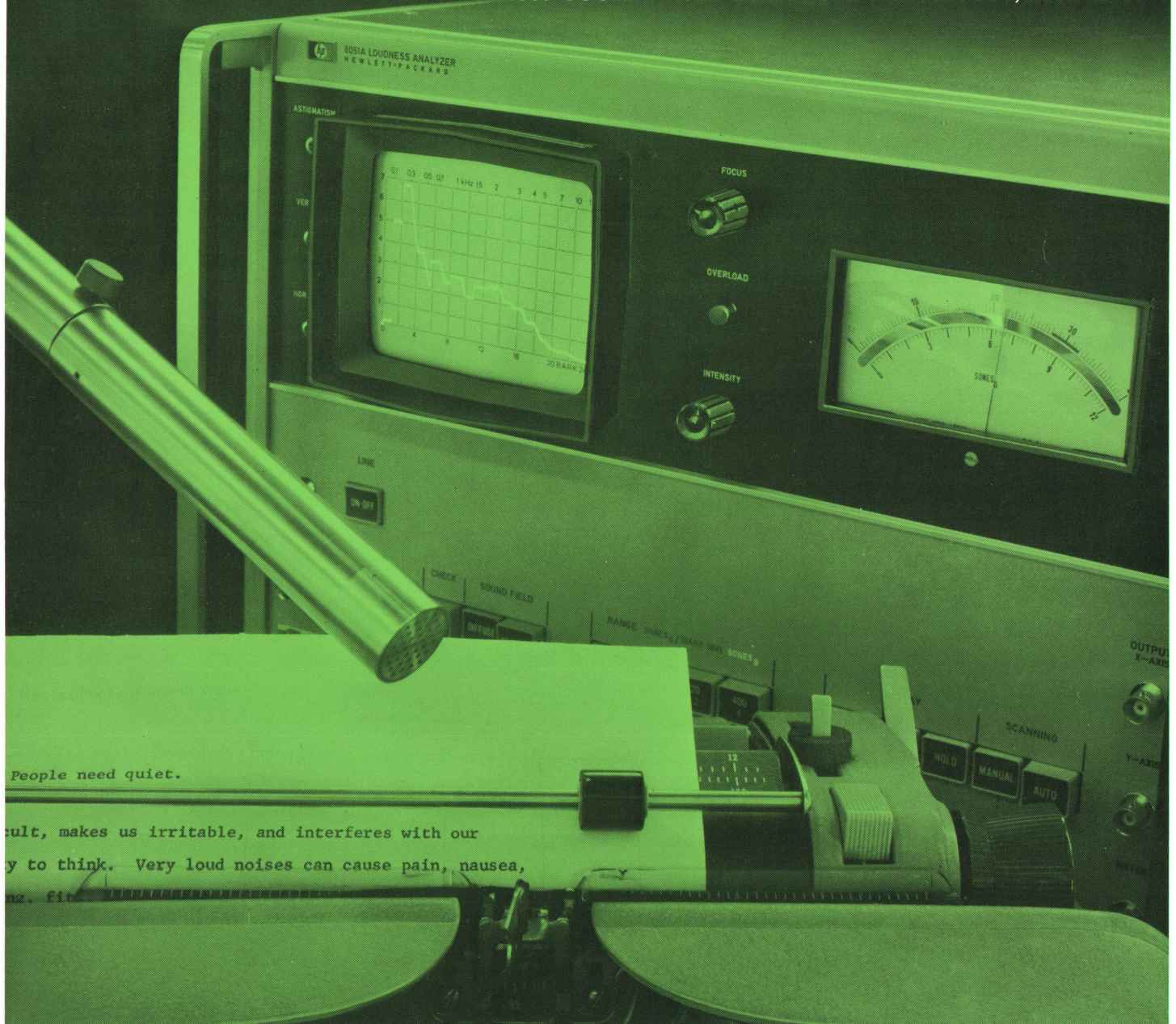
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LOUDNESS EVALUATION

Effective noise abatement calls for instruments that can measure loudness. But loudness is subjective, and instruments aren't like people.

By Wolfgang E. Ohme

People need quiet.

Prolonged loud noise damages hearing, makes sleep difficult, makes us irritable, and interferes with our ability to think. Very loud noises can cause pain, nausea, fainting, fits, psychosis, or death.

The sad truth is that our environment is getting noisier all the time.

If this bothers you, you aren't alone. There is a great deal of effort being expended these days to reduce the amount of objectionable sound that bombards us. In these noise abatement efforts, the measurement of loudness plays a critical role.

Not all sounds are noise, of course. Many sounds carry information which is useful or essential for our life. Speech and music are two examples; the sound of an automobile horn is another.

Sometimes it isn't easy to decide whether a sound is information or noise. Often it is both. For example, the sound of a machine can be considered an information-carrying sound because it tells the machinist whether or not his machine is functioning properly. But for his neighbor who is operating another machine, this sound is noise — it carries no useful information.

Most of the everyday sounds we hear are noise to us; yet many of them carry information for someone else. It is a function of society to establish limits to keep noise to a minimum while insuring that information-carrying sounds are audible to those who need to hear them.

If we want to define such limits we have to be able to measure them. This turns out to be a difficult task because the yardstick that must be applied is the subjective sensation of loudness, that is, loudness as it is experienced by people. This sensation seems to involve complicated physiological and psychological mechanisms.

A good loudness meter would have to imitate many unique properties of the human ear. These properties have been extensively investigated by a great number of scientists. However, we still do not have a very good understanding of the physiological processes underlying many of them; our knowledge of these properties is only empirical. We still can't make a complete model of the ear. Nevertheless, we have learned to make fairly accurate models of the loudness-sensing function of the ear.

A number of approximations have been formulated for computing a quantity proportional to loudness, using the results of more-or-less detailed analyses of the noise to be evaluated. We shall discuss three of these methods in this article. Two of these are the calculation methods of Zwicker and Stevens; the third is the comparatively simple sound-level meter. The methods of Zwicker and Stevens have been internationally accepted in Recommendation 532 of the ISO (International Organization for Standardization). Except for some recent refinements, the sound level meter is described in ISO Recommendations 123 and 179.

Some scientists believe, with good reason, that loudness is not a completely satisfactory measure of how much a sound will disturb a person. Attempts have been made to define a better measure, called annoyance. So far, these attempts have not met with much success, chiefly because of the large number of unknown psychological factors that contribute to the effect of any sound on any individual at any time. These factors include such things as a person's past history, his present state of mind, what he is trying to do at the moment, and so on. One definition of annoyance that has found some acceptance is Kryter's 'perceived noise' concept (ISO R 507, later modified), which uses a method similar to Stevens' loud-

ness-computing method to arrive at annoyance in PNdB. Kryter's method is designed primarily for the type of noise produced by jet aircraft. At present, this method is in a state of flux, and no one is certain what its final form will be. Some experts feel that a modification of the simple sound level meter should give adequate results for jet aircraft. It appears, therefore, that until our understanding of the psychological effects of sound improves greatly, the only reasonably objective measure of the disturbing power of a sound is its loudness.

Sound Pressure and Sound Pressure Level

Sound at a particular point is a rapid variation in the pressure at that point around a steady-state value. In air, the steady-state pressure is atmospheric pressure (which actually changes, but slowly enough to be considered constant compared to the rapid pressure variations of sound).

Sound pressure is measured in the same units as atmospheric pressure. It is an alternating quantity, and usually the term 'sound pressure' refers to its rms value.

At a frequency of 1 kHz, a sound with an rms pressure of $2 \times 10^{-4} \mu$bar*, or about 2×10^{-10} atmosphere, is just below the threshold of hearing for good ears; that is, a sound of this magnitude is inaudible, but slightly larger sound pressures can barely be heard. This demonstrates the amazing sensitivity of the human ear — it can detect variations in atmospheric pressure as small as a few parts in 10^{10}.

Another of the remarkable properties of the human ear is its large dynamic range. At 1 kHz, it can hear sounds as small as about $2 \times 10^{-4} \mu$bar, and at the other end of the sound-pressure scale, it can accommodate sound pressures as high as 200μbar without becoming overloaded. Bigger sounds, say $2,000 \mu$bar, are physically painful.

Because the dynamic range of the ear is so large it is common practice to use a logarithmic scale for sound pressure. A reference value of $2 \times 10^{-4} \mu$bar, approximately the threshold of hearing at 1 kHz, has been agreed upon. Rms sound pressure is commonly expressed in dB above $2 \times 10^{-4} \mu$bar and referred to as *sound pressure level*. Mathematically, if p is rms sound pressure and P is sound pressure level, then

$$P = 20 \log_{10} \frac{p}{p_0} \text{dB}$$

where $p_0 = 2 \times 10^{-4} \mu$bar.

In terms of sound pressure level, then, the ear's dynamic range is about 120 dB. Not many electronic instruments can match this.

* One μbar equals one dyne per square centimeter or 0.1 newtons per square meter. A human speaker at a distance of one meter generates a sound pressure of about one μbar.

Sound Fields

Sound in its environment can be thought of as a field, just as electromagnetic waves are fields. Three common types of sound fields are the plane sound field, the spherical sound field, and the diffuse sound field.

Sound in a homogeneous space propagates outward from a source in all directions and consequently forms a spherical field. In a spherical field, the sound pressure decreases with the square of the distance from the source.

When a microphone is relatively far away from a source the sound field may appear to be a plane field, in which the sound pressure is constant in any plane perpendicular to the direction of propagation.

If sound is generated in a room, sound waves are reflected from the walls, and a directional sound field can only be found very close to the source. Further from the source, sound approaches any point uniformly and randomly from all directions. Thus the sound field is diffuse. Such a field would be found in a factory if the nearest machine were not too close.

It is often important to know whether the sound field in an area is approximately plane or diffuse. If it is plane, directional microphones can be used with advantage to measure it; if it is diffuse, omnidirectional microphones are needed. Often a field will be partly plane and partly diffuse. In a factory, for example, a machinist is in the directional sound field of his own machine but in the diffuse sound field of noisy machines in the distance.

The transition from a directional sound field to a diffuse sound field in a room is characterized by a critical radius, which can be estimated as follows:

$$r_G = 0.14 \sqrt{\bar{a}A},$$

where $\bar{a}$ is the absorption coefficient of the walls and A is the surface area of the walls, floor, and ceiling. In an average factory $\bar{a}$ is between 0.05 and 0.2. In normal rooms $\bar{a}$ is between 0.1 and 0.3. The change from a directional or plane field to a diffuse field can be considered to occur at a distance r_G from the sound source.

In loudness measurements, two types of field are usually considered. One is the *diffuse* field. The other is a plane sound field which approaches the hearer from the front, head on; this field is called a *frontal* sound field.

Loudness Level

Since loudness is a subjective quantity the primary instrument for measuring it can only be a human observer. To determine whether one sound is louder, equally loud, or less loud than another, we would have to let a statistically significant number of people compare the sounds and then average their opinions. Similarly, to de-

termine how loud a sound is, we would have to choose a standard sound and have a significant number of people compare the unknown with the standard.

In acoustics the accepted standard is a pure 1 kHz tone or narrow-band noise centered at 1 kHz. The *loudness level* of any sound is defined as the sound pressure level of a standard sound which appears to a significant number of observers to be as loud as the unknown. Loudness level is measured in *phons,* the loudness level of any sound in phons being equal to the sound pressure level in dB of an equally loud standard sound. Thus a sound which is judged to be as loud as a 40 dB 1 kHz tone has a loudness level $L = 40$ phons.

Loudness

Although the logarithmic phon scale covers the large dynamic range of the ear (120 dB) conveniently, it does not fit a subjective loudness scale. A factor of two in loudness does not correspond to double the number of phons. Over most of the audible range, that is, for loudness levels of 40 phons and greater, the corresponding increment is 10 phons. This is an empirical fact; why loudness should be different from physical quantities like voltage, for which a factor of two corresponds to 6 dB, is not fully understood.

It is also difficult to add loudnesses in phons. If, for instance, we produce one tone at 200 Hz with a loudness level of 70 phons, and another at 4 kHz with the same loudness level, it would be convenient if both tones together would yield a loudness level of 140 phons. Unfortunately, this doesn't happen. The two tones actually are perceived as a loudness level of 80 phons.

In an effort to obtain a quantity proportional to the intensity of the loudness sensation, a loudness scale was defined in which the unit of loudness is called a *sone*. One sone corresponds to a loudness level of 40 phons. For loudness levels of 40 phons or greater, the relationship between the numerical values of loudness level L (in phons) and loudness S (in sones) is given by

$$S = 2^{(L-40)/10} \qquad (1)$$

(ISO Recommendation R 131).

Table I compares the loudnesses (sones) and loudness levels (phons) of several common sounds. Notice that the loudness scale in sones corresponds fairly closely to our subjective sensation of loudness. We feel, as a matter of experience, that a speaker in an auditorium speaks about four times as loudly as someone who talks quietly with us in normal conversation. It is more meaningful to state that a jet aircraft at takeoff is about 50 times as loud as our conversation than to state that the jet aircraft generates 120 phons in contrast to 60 phons generated in ordinary conversation.
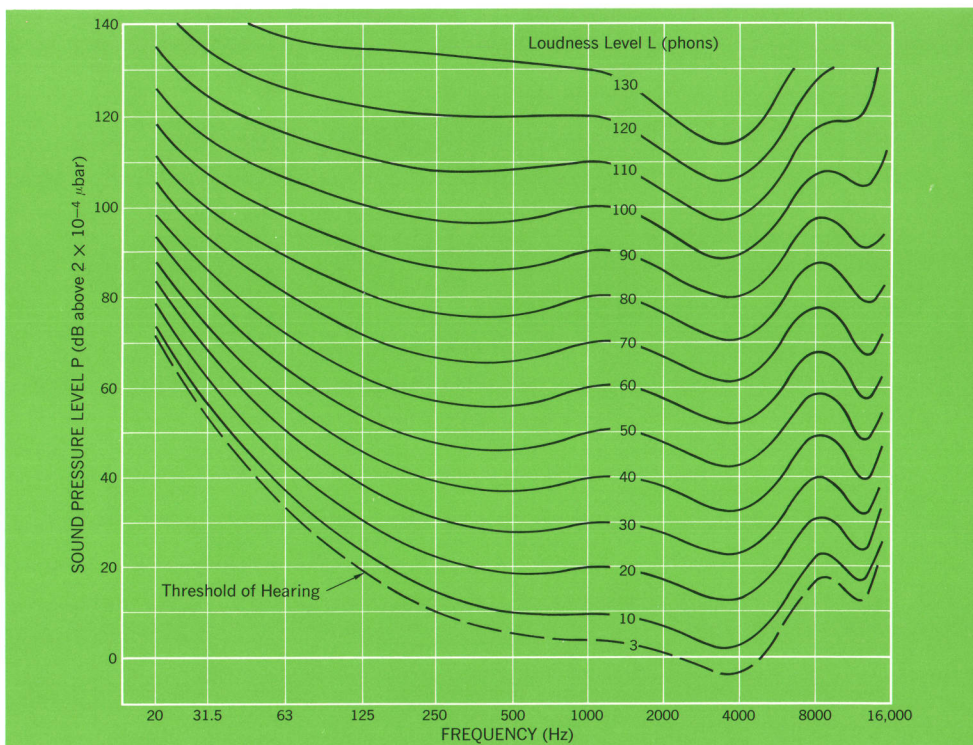

Fig. 1. *Curves of equal loudness level for pure tones in frontal sound field, according to ISO Recommendation 226. These curves show how frequency response of the human ear varies with loudness level.*

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Table I

Loudness Level (phons)		Loudness (sones)
140	Threshold of pain	1024
120	Jet aircraft	256
100	Truck	64
80	Orator	16
60	Low conversation	4
40	Quiet room	1
20	Rustling of leaves	
3	Hearing threshold	

Fig. 2. *Difference between equal-loudness-level contours in frontal diffuse sound fields, according to ISO R 454.*

Loudness — a Function of Frequency

The loudness level of a 1 kHz tone is the same as its sound pressure level. This would also be true of pure tones of other frequencies if perception were constant with frequency. However, it is not. The loudness level of any other sound (in phons) is not, in general, equal to its sound pressure level (in dB). For example, if a large number of observers compare a 100 Hz tone with a 1 kHz tone, they will judge the two to be equally loud only when the 100 Hz tone has a higher sound pressure level than the 1 kHz tone. The frequency response of the ear is not flat.

Although the subjective sensation of loudness differs from person to person, normal ears seem to agree within a few dB, at least for the young, male subjects who have participated in most subjective tests. Hence it is possible to draw curves or contours of equal loudness level for normal ears, as shown in Fig. 1.

Equal loudness level contours were first published in 1933 by Fletcher and Munson. The slightly modified form of their curves shown in Fig. 1 is now universally accepted as reference data (ISO Recommendation 226).

The curves of Fig. 1 are for pure tones in a frontal sound field. They show, for example, that a 40 phon 100 Hz tone has a sound pressure level of 50 dB, but an equally loud 40 phon 1 kHz tone has a sound pressure level of only 40 dB.

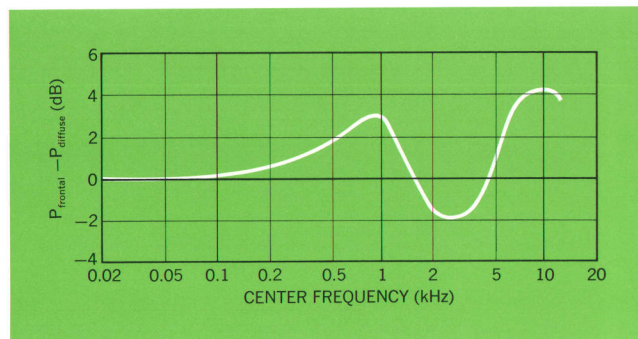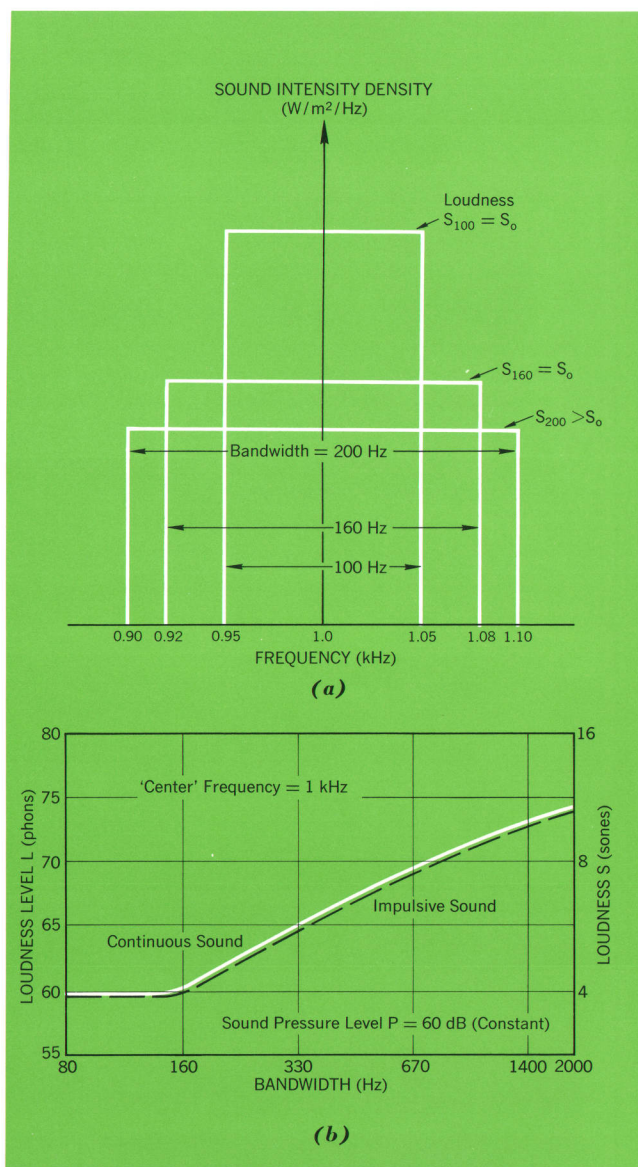The 3 phon curve is just above the threshold of hearing for normal ears.

(a)

(b)

Fig. 3. *Effect of bandwidth on loudness is demonstrated by holding sound pressure level constant at 60 dB, keeping center frequency fixed at 1 kHz, and increasing bandwidth.*

(a) As bandwidth increases, sound intensity density (W/m²/Hz) must decrease to keep rms sound pressure constant. Area of each rectangle is sound intensity of corresponding sound, which is proportional to square of rms sound pressure. All rectangles have same area.

(b) As bandwidth increases, keeping sound pressure level constant as in (a), loudness level is not affected until bandwidth exceeds a critical value. Different center frequencies have different critical bandwidths. At 1 kHz, critical bandwidth is 160 Hz.
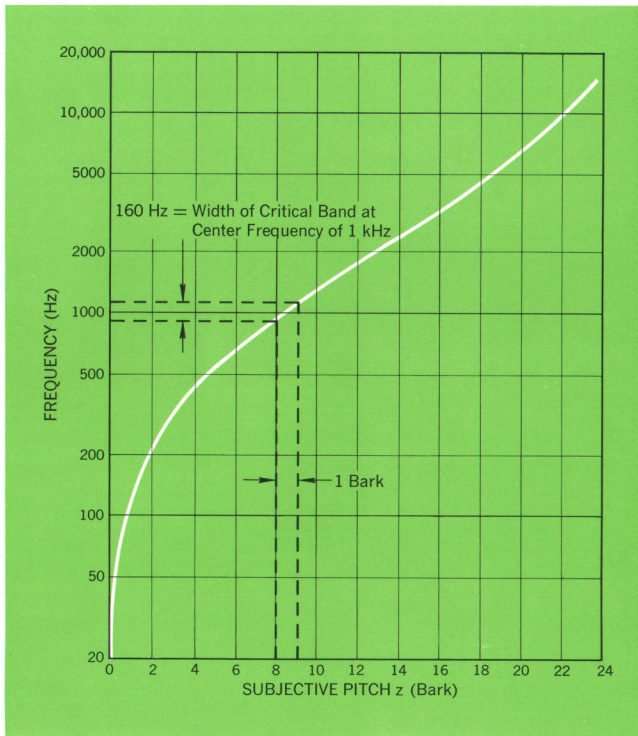
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Fig. 4. *Subjective pitch scale is different from frequency scale; in subjective pitch, half way between 0 and 4 kHz is about 1 kHz, not 2 kHz. For some as-yet-unkown reason, critical bandwidths at all center frequencies correspond to subjective pitch intervals of the same width. This width is defined as one Bark. (Critical bands are defined in Fig. 3.)*
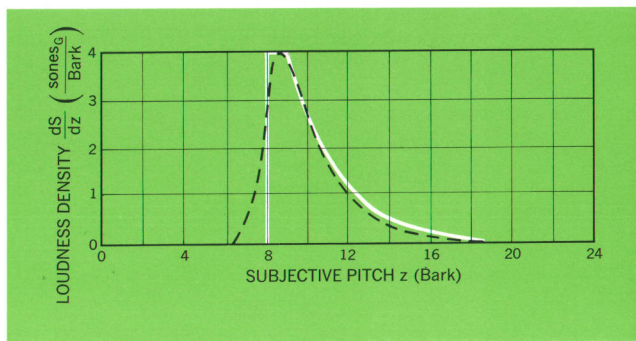

Fig. 5. *Loudness density is a quantity which, when integrated over subjective pitch, gives total loudness in sones_G. (The subscript G indicates a value calculated from a critical-band analysis, as opposed to a subjectively measured value.) This diagram shows a masking curve (dashed line) for a pure 1 kHz tone which has a loudness of 13 sones_G. The 1 kHz tone excites nerves in the ear as shown by the dashed line. The solid curve is an approximation which has been found to give good results in loudness measurements. The area under each curve is 13 sones_G.*

Notice that the curves converge at low frequencies, but are approximately parallel between 1 and 10 kHz. This means that the ear's frequency response is a function not only of frequency but also of level. Therefore it can be simulated only with networks which are nonlinear with respect to both frequency and amplitude.

Curves of equal loudness level for a diffuse sound field can't be measured using pure tones, because it is difficult to set up a diffuse field using pure tones. Pure tones are likely to bounce off walls and nearby objects and produce standing-wave patterns, whereas sound in a diffuse field is supposed to be uniform in all directions. However, diffuse-field loudness comparisons can be carried out with consistent results using frequency-modulated tones or narrow-band noise.

Differences in sound pressure levels necessary to give the same sensation of loudness in a diffuse field as in a plane field were standardized in ISO Recommendation 454. Using these differences (see Fig. 2), curves of equal loudness level for the diffuse sound field can be calculated from those for the plane field.

Loudness — a Function of Bandwidth

To human ears, broadband sounds, like those of jet aircraft, seem much louder than pure tones or narrow-band noise having the same sound pressure level. Fig. 3 illustrates this effect for band-limited noise having a center frequency of 1 kHz.

Fig. 3(a) is a series of sound intensity density spectra for bandwidths of 100 Hz, 160 Hz, and 200 Hz. All three spectra have the same area, so all three noises have the same sound intensity (sound power per unit area). This means that all three noises have the same sound pressure level. But all three noises are not equally loud.

If the loudness of the noise which has 100 Hz bandwidth is S_0, then the loudness of the noise which has 160 Hz bandwidth is also S_0. But the loudness of the noise which has 200 Hz bandwidth is *greater* than S_0.

Fig. 3(b) shows what increasing bandwidth does to the loudness of noise having a center frequency of 1 kHz and a constant sound pressure level of 60 dB. Up to a *critical bandwidth* of 160 Hz, the subjective loudness is constant. Beyond that point, however, there is a marked increase in loudness. At a bandwidth of 2 kHz the loudness level L has increased from 60 phons to 74 phons. Loudness S has increased by a factor of 2.5.

Similar investigations, using different center frequencies, yield different critical bandwidths. At a center frequency of 200 Hz the critical bandwidth is approximately 100 Hz. At 5 kHz it is about 1 kHz.

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We cannot account for the effect of bandwidth on loudness with any broadband measurement. Accurate loudness measurements can be made only by taking into account the spectral distributions of sounds being analyzed. The necessary degree of resolution in the spectrum analysis is clear from Fig. 3(b). Obviously, we need no filter having a bandwidth narrower than a critical bandwidth, because for narrower bandwidths the spectral distribution of the sound doesn't influence loudness. Conversely, no filter should have a bandwidth wider than the critical bandwidth corresponding to its center frequency; if it does, the measured loudness will be incorrect.

Twenty-four filters, each having a bandwidth equal to the empirically determined critical bandwidth for its center frequency, will cover the audio range between 20 Hz and 15,500 Hz.

Subjective Pitch

The human ear's critical bands seem to be related to another property of the ear, namely, subjective pitch. Subjective pitch tells us how our ears compare the frequencies of different sounds.

If an average untrained observer — not a musician or a piano tuner — were first allowed to listen to two tones, say a very-low-frequency tone and a 4 kHz tone, and then were asked to tune an oscillator until he heard a tone that fell exactly half way between the first two tones, he would not pick something around 2 kHz. Instead, he would pick a tone having a frequency of about 1 kHz. In subjective pitch, then, 1 kHz is halfway between 0 and 4 kHz. The unit of subjective pitch is the *mel;* 0 to 2400 mel span the frequency range 0 to 16 kHz.

Remarkably enough, it turns out that a subjective pitch interval of approximately 100 mel located anywhere in the audio range corresponds to the width of a critical band at that point! Probably, the same mechanism in the ear is responsible both for critical bands and for subjective pitch. However, our understanding of the ear is still not good enough to allow us to identify this mechanism.

In loudness measurements, the frequency scale most commonly used is linear in subjective pitch z. However, the mel is not used. Instead, the width of a critical band is defined as one *Bark*. Accordingly, the audio range comprises 24 Bark.

Fig. 4 shows how subjective pitch, in Bark, is related to frequency.

Loudness — a Function of Proximity

Two sounds presented to the ear simultaneously produce a sensation of loudness which is larger than that
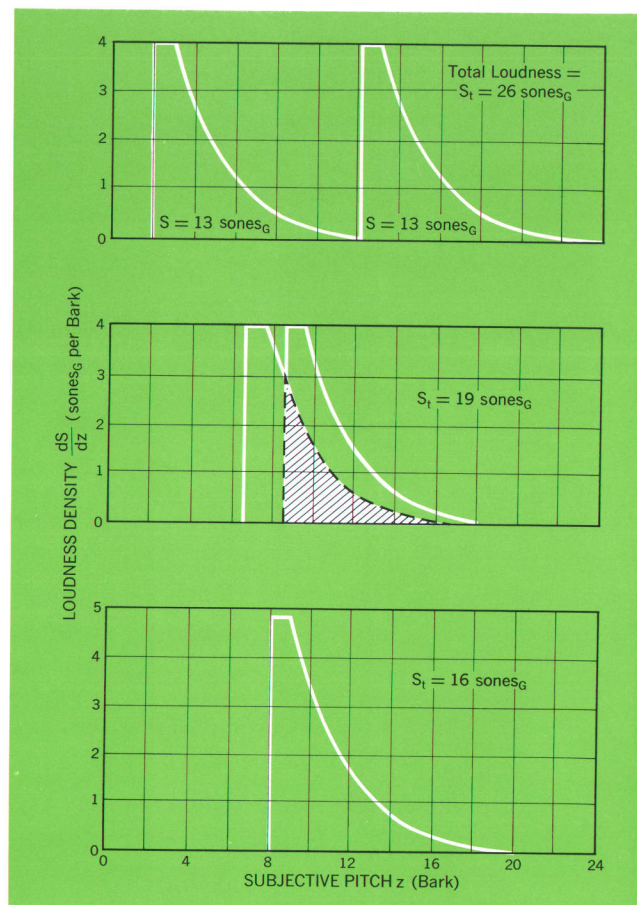

Fig. 6. *Masking occurs when two or more sounds excite the same nerves in the ear. Two sounds of equal loudness are twice as loud as one sound if they are separated widely in frequency (top). If their masking curves overlap (center), total loudness is given by area under solid curve and is less than twice the area under one curve. If both sounds fall in the same critical band and are incoherent (bottom), their loudness level increases by only 3 phons.*

produced by either of them alone. Take for example a 200 Hz tone having a loudness level of 70 phons and a 4 kHz tone, also having a loudness of 70 phons. If two sounds are as widely separated in frequency as these two, their partial loudnesses simply add to form the total loudness. The loudness corresponding to a loudness level of 70 phons is 8 sones. If two partial loudnesses of 8 sones each occur simultaneously the total loudness is 16 sones, and the loudness level is 80 phons.

This simple summation of partial loudnesses can only be carried out if the individual sounds are separated widely in frequency. The closer they are in frequency the more they influence each other, and total loudness may
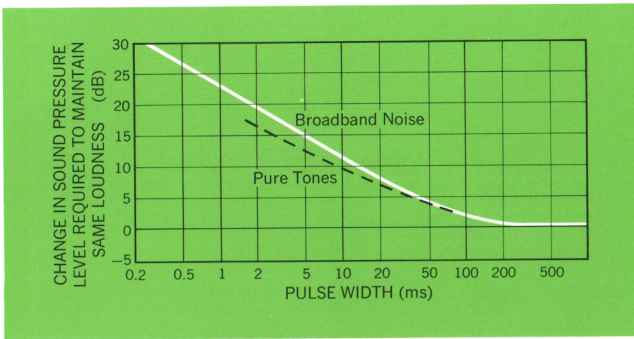
Fig. 7. *As duration (pulse width) of a sound decreases, its sound pressure level must increase to maintain same loudness.*

not be quite so large as the sum of the partial loudnesses. This effect is called partial masking. In the extreme case, partial masking becomes total masking, wherein a strong sound renders a lower-level sound completely inaudible. When total masking occurs, low-level sound components cannot be heard at all and do not contribute to loudness.

The partial masking of tones cannot be understood in terms of level and frequency because pure tones represented by spectral lines cannot influence each other. Investigations on the ear have shown, however, that even pure tones or narrow-band noise excite nerves in the ear that correspond to a wide range of frequencies. Masking occurs because the ear treats sounds in an 'OR' fashion — when two sounds excite the same nerves, the ear hears only the larger sound in that frequency range.

Loudness Density

The manner in which pure tones or narrow-band sounds excite nerves in the ear corresponding to many frequencies can be expressed quantitatively in terms of a parameter called *loudness density*. Loudness density is defined as a function of subjective pitch; when loudness density is integrated over subjective pitch, the result is total loudness. If integration over subjective pitch is to yield total loudness then loudness density must be the differential quotient of loudness S over subjective pitch z, i.e. $\dfrac{dS}{dz}$.

Loudness density $\dfrac{dS}{dz}$ has the dimensions of sones$_G$ per Bark. The subscript G indicates that this loudness is calculated in terms of critical bands, not subjectively measured.

Loudness density versus subjective pitch for a 1 kHz tone at a sound level of 77 dB is shown in Fig. 5 as a dashed curve. This function has been established in many masking experiments. The area under the curve, that is

$$S = \int_{z=0}^{z=24\ \text{Bark}} \frac{dS}{dz}\, dz,$$

is 13 sones$_G$. This was to be expected because according to equation (1), 77 phons corresponds to 13 sones.

For most applications, especially for calculating loudness, it is sufficient to approximate the dashed curve by the curve shown in Fig. 5 as a solid line. The horizontal portion at the top of the solid curve has a width of 1 Bark. The height of this horizontal portion can be called the *band loudness density*. Band loudness densities and the shapes of the tails of the approximate curves have been determined empirically for various sound pressure levels and frequency bands.

Masking

Mutual partial masking of two tones can be represented and explained very effectively in terms of loudness density. The three examples in Fig. 6 show the loudness density functions of two 77-phon tones in different relationships. Fig. 6(a) the distance between the tones is larger than 10 critical bands. In Fig. 6(b) it is larger than two critical bands, and in Fig. 6(c) it is less than one critical band. In the first case total loudness is the sum of the individual loudnesses, that is, 26 sones$_G$. When the loudness density curves overlap, as in Fig. 6(b), we get a total area which is smaller than the sum of the individual areas. This is partial masking. In our example, the two sounds now have a loudness of only about 19 sones$_G$.

When both tones fall into the same critical band, Fig. 6(c), and yet are of different frequencies, the sound pressure level in that band increases by 3 dB to 80 dB. The area under the loudness density curve increases by an amount which corresponds to an increase in loudness level by 3 phons, that is, by roughly 20%. Thus the total loudness is only 16 sones$_G$.

Loudness of Impulses

The sounds heard in everyday life are not all uniform. Many, like bangs and rattling sounds, change rapidly with time. The dependence of the loudness of a sound on its duration can be represented by a curve of equal loudness as a function of pulse width (Fig. 7). Subjective measurements have yielded similar results for short bursts of pure tones (dotted line in Fig. 7) and for short bursts of broadband noise (solid line in Fig. 7). Loudness is independent of duration for large pulse widths. Only when the pulse width drops below about 100 ms does the level of a pulse have to be increased to yield the same loudness.

Then, when the pulse width decreases by a factor of two, the sound pressure level of the impulse must increase by 3 dB to give the same loudness. In other words, the ear appears to have a time constant of about 100 ms.

The dependence of loudness on bandwidth was shown in Fig. 3. But are these laws also valid for short sounds, i.e., for sound impulses? It turns out that they are. The dashed curve in Fig. 3(b) is for band-limited noise impulses of 5 ms duration. The loudness level of these short impulses increases with bandwidth once the width of a critical band is exceeded in the same way that it does for continuous sound. Thus the laws describing loudness in terms of critical bands and loudness density are valid for impulsive noise. However, for arbitrary impulses and time-dependent sounds, the critical-band levels must be measured with the temporal weighting described in the preceding paragraph. In practical terms this means that the band levels should be measured with rms detectors having integration times of about 100 ms.

Sound Level Meter

The first attempt at measuring loudness was made quite some time ago in the form of the *sound-level meter*. In this instrument, sound pressure is transformed into voltage by a microphone; a weighting network shapes the voltage to account for the frequency response of the ear; and a quasi-rms voltmeter with a logarithmic scale indicates the weighted sound-pressure level.

Until recently, sound-level meters had three possible weighting curves. These are called A, B, and C, and are specified in ISO R 123 and ISO R 179. Each curve is used for a particular range of sound pressure levels, and results are given in dB (A), dB (B), or dB (C).

Recently a new weighting curve, the N curve, has been proposed for measuring the loudness of broadband noise, like that of jet aircraft. The N curve seems to be gaining some support among acoustical experts, but so far no international standards have been written for it.

Fig. 8 shows the four weighting curves.

Sound-level meters have the advantage of simplicity, but begin to give us problems when we expose them to more than one tone at a time. As we have seen, one broadband network cannot weight a high-level low-frequency tone and a low-level high-frequency tone properly at the same time. In one typical case, for example, a sound-level meter gave a reading of 110 dB (A) for a 1 kHz tone, and the same reading for a broadband signal consisting of an unsymmetrical square wave plus a sine wave. The actual loudnesses, computed by the more accurate Zwicker method (described later in this article)

Fig. 8. *International standard A, B, and C weighting curves for sound-level meters. Each curve is used for a particular range of sound pressure levels. Also shown is the proposed N weighting curve for jet aircraft noise.*
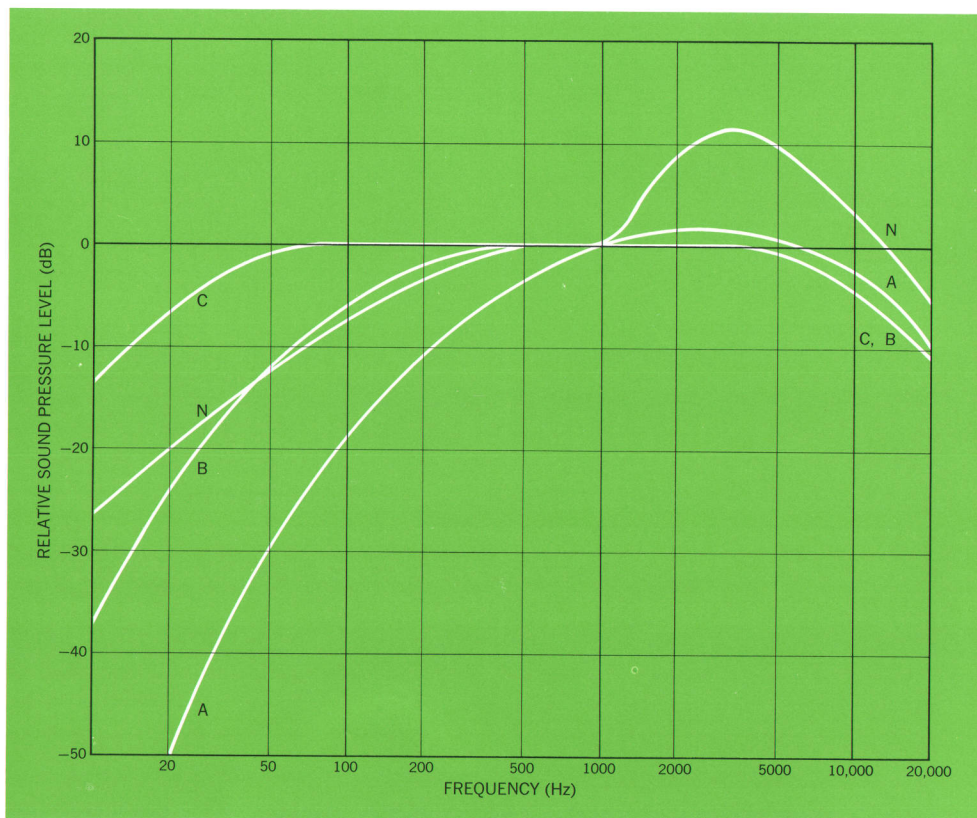
were 128 sones$_G$ for the pure tone, and 340 sones$_G$ for the broadband signal. Corresponding loudness levels are 110 phons$_G$ and 124 phons$_G$, respectively. The difference of 14 phons$_G$ represents an error in the dB (A) reading. Sound-level meters are also unable to account for masking, and in their present form are not useful for measuring the loudness of impulsive sounds.

Stevens' Procedure for Calculating Loudness

Addition of partial loudnesses is the fundamental notion in all of the known procedures for calculating loudness.

In Stevens' procedure, partial loudnesses are determined by means of families of curves (Fig. 9) from sound levels which are measured in octave, ½-octave, or ⅓-octave frequency bands. These partial loudnesses are
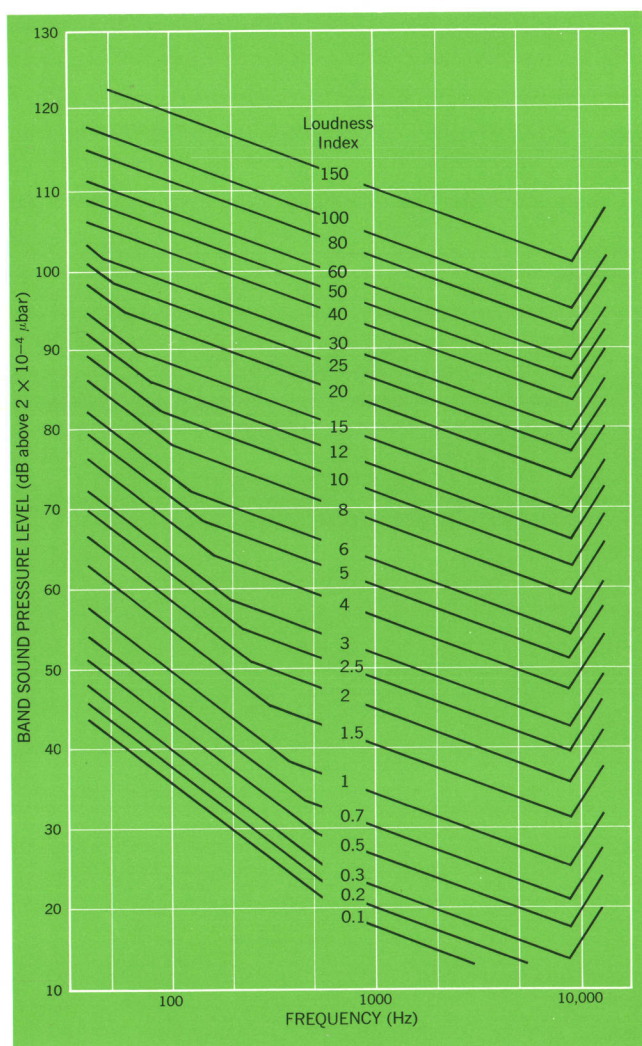
Fig. 9. *Curves for determining loudness indexes used in Stevens' method of calculating loudness.*

called indexes. Partial masking is taken into account very generally by multiplying all loudness indexes, except the one with the largest number, by a factor smaller than 1. The partial loudnesses produced in such a way are added to the largest partial loudness to give total loudness. Total loudness is given by the formula:

$$S = S_m + F\left(\sum S - S_m\right),$$

where S_m is the maximum loudness index and $\sum S$ is the sum of all loudness indexes. F is the factor which takes masking into account. Its value is 0.3 for octave, 0.2 for ½-octave, and 0.15 for ⅓-octave frequency bands.

Stevens' procedure is standardized in ISO R 532, Method A.

Zwicker's Procedure for Calculating Loudness

In Zwicker's procedure (ISO R 532, Method B), the frequency range between 45 Hz and 14 kHz is divided into bands which approximate critical bands. Filters are used to break the sound into its components, and the loudness in each band is measured. The measured loudnesses are then plotted on a diagram which automatically accounts for partial masking (Fig. 10).

Because filters with the widths of critical bands are not normally available, the procedure has been modified to use ⅓-octave filters. Practically speaking, this doesn't introduce any inaccuracy into the results. Subjective measurements often disagree with each other by ±20% or more, and this is much greater than the amount by which ⅓-octave analyses differ from critical-band analyses. Actually, between 280 Hz and 14 kHz, the widths of critical bands are quite close to ⅓ octaves.

Below 280 Hz, ISO R 532 requires that sounds be grouped into two octave-wide bands and one ⅔-octave band. Thus the audio range is spanned by 20 filters, two having octave bandwidths, one having ⅔-octave bandwidth, and 17 having ⅓-octave bandwidths. The differences in band loudness densities which result from using ⅓-octave filters instead of critical-band filters are taken into account by small changes in the scales on the Zwicker diagram.

There are eight different Zwicker diagrams, four for frontal sound fields of different maximum sound pressure levels, and four for diffuse fields of different levels. Once the proper diagram has been chosen, measured ⅓-octave sound pressure levels are placed on the diagram as horizontal lines between the frequency limits of the appropriate bands. In the same way, the levels calculated from ⅓-octave levels in the three lowest bands are entered. This gives 20 horizontal lines on the diagram. At the
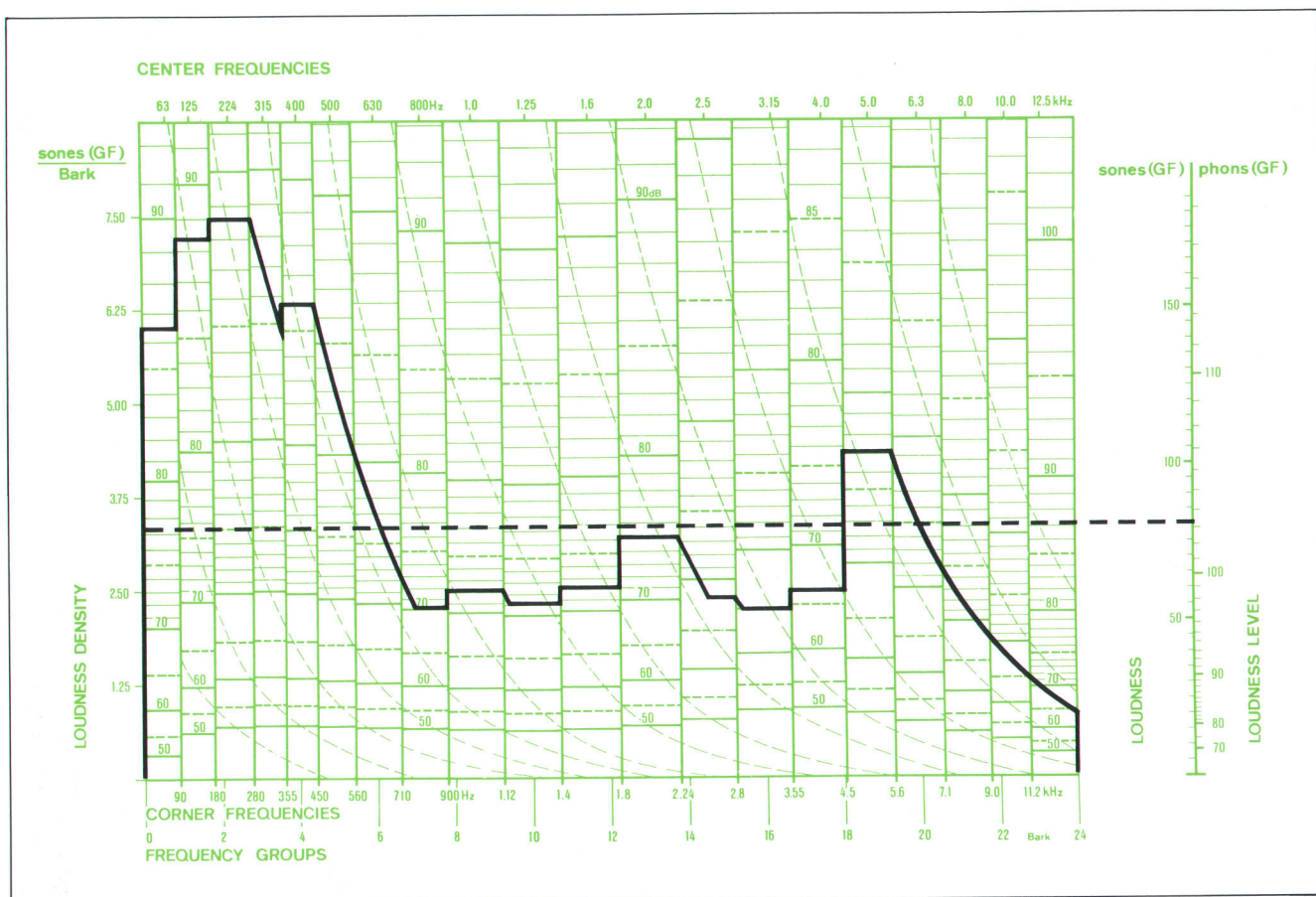
Fig. 10. *Zwicker diagram with curve representing analysis of a sound. Total loudness is given by area under curve. Sound-pressure levels measured in each frequency band are entered as horizontal lines, then connected by following dashed auxiliary lines which account for masking. Area under curve can usually be found quite accurately by eye, simply by moving a ruler or the equivalent up and down until the areas between the ruler and the curve seem to be equal above and below the ruler. This is illustrated by the dashed horizontal line. Loudness or loudness level can then be read where the ruler (dashed line) intersects the scale at right; in this case loudness is 79 sones$_{GF}$. (The subscript GF indicates a critical-band analysis and a frontal sound field.)*

high-frequency limit of each band, other lines are drawn, parallel to the dashed auxiliary lines contained in the diagram. Thus we get a loudness density-subjective pitch diagram, the total area of which gives total loudness. An example is given in Fig. 10. The loudness of the sound shown in Fig. 10 has been calculated by this procedure to be:

$$S_{GF} = 79 \text{ sones}_{GF} \text{ or } L_{GF} = 103 \text{ phons}_{GF}.$$

(The subscript GF indicates a critical-band analysis and a frontal sound field.)

Zwicker's procedure is as valid for impulsive sounds as it is for uniform sounds. If an rms voltmeter is used to indicate the critical-band levels or the ⅓-octave levels for impulsive sounds, it will have to have a high crest-factor capability and an integration time of approximately 100 ms. This is roughly the time constant of the ear. ∎
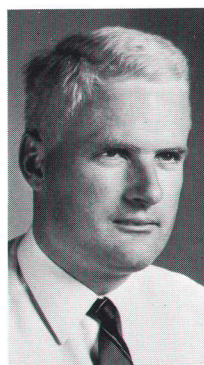
Wolfgang E. Ohme

Wolfgang Ohme graduated from the Technical University of Stuttgart with the degree of Diplom-Ingenieur in electrical engineering. Before joining HP in 1962, he was a faculty member of that school for nearly five years, teaching courses in network theory and supervising graduate work.

Wolfgang was the first member of the research and development department of HP GmbH in Böblingen, West Germany. In his present position of engineering manager he is responsible for projects concerned with acoustical instrumentation, pulse techniques, and medical electronics.

Wolfgang is a member of NTG, a national professional society for communication engineers.

Automatic Loudness Analysis

Measuring the subjective sensation of loudness is easy if you have one of these calibrated electronic ears.

By Heinz Blässer and Helmut Finckh

HOW LOUD IS THE SOUND OF A TYPEWRITER KEY striking the paper? How can it be made quieter?

If you make typewriters, you need answers to questions like these. If you make cars, airplanes, or machines, or if you operate an office, a factory, a hotel, or a radio station, you probably need answers to similar questions about your own noise problems.

But getting answers to questions about loudness has been difficult, to say the least. Until rather recently, much too little was known about how the ear translated sound pressure into loudness. Early sound-level meters attempted to measure loudness by measuring the level of a frequency-weighted sound pressure. They gave good results for continuous, narrow-band sounds, but were often in error by up to 20 dB for wideband or impulse sounds, like those produced by jet aircraft or office machines. This could mean, for example, that when a sound-level meter says a rattling noise has the same loudness as a continuous tone, the ear might actually perceive the rattling noise to be four times as loud as the pure tone. This is a 400% error!

Recent research has given us a much better understanding of how the human ear works. One result of this research is a new instrument that responds to the loudness of sounds in very much the same way that the ear does.

The new loudness analyzer described in this article gives data which correspond closely to the subjective sensation of loudness. It does this by simulating the known characteristics of the human ear according to Zwicker's method, which is described in ISO Recommendation 532 and in the article beginning on page 2. The analyzer works for wideband or narrow-band, continuous or impulsive sounds. It can even analyze a single-shot sound — a single typewriter stroke, for example.

The analyzer takes inputs from a microphone or a tape recorder and makes a continuous Zwicker analysis of them. It displays the resulting Zwicker diagram (a plot of loudness density versus subjective pitch) on a CRT, showing how the input components in each of 20 frequency bands contribute to the total loudness (Fig. 1). A new plot is made every 25 ms, so that even transient sounds can be analyzed conveniently. The analyzer can also be instructed to hold its most recent loudness analysis for several minutes. This allows the analysis of a changing sound to be frozen at any desired time and held long enough for it to be recorded or photographed. Using this feature, the analyzer can make Zwicker plots automatically on an X-Y recorder [see Fig. 2(c)]. Sound pressure levels in each channel can be read from the special Zwicker recorder paper.

Total loudness of a sound, that is, the integral of the Zwicker diagram, is also computed by the analyzer and displayed on a meter (Fig. 1). A recorder

Fig. 1. *New HP Model 8051A Loudness Analyzer measures loudness of narrow-band or wideband, continuous or impulsive sounds. Analyzer makes Zwicker-type analysis, gives results which agree well with subjective sensation of loudness. CRT shows loudness density versus subjective pitch; meter shows total loudness.*

(a)

(b)

08051 - 90102
HEWLETT · PACKARD

8051A RANGE: 120 sones$_G$
SOUND FIELD: FRONTAL

(c)

Fig. 2. *Typical analysis of the loudness of a single typewriter stroke. (a) Sound pressure waveform has low-frequency components produced by carriage, and large, high-frequency, two-millisecond transient produced by key hitting paper. (b) Loudness analyzer display for typewriter stroke. Total loudness was 100 sones$_G$. (c) Zwicker diagram for typewriter stroke, produced by loudness analyzer while CRT display (b) was being photographed.*

output is provided for recording loudness versus time.

The analyzer has four measurement ranges which accommodate sounds with loudnesses of 1 to 400 sones$_G$, equivalent to loudness levels of 40 to 127 phons$_G$.* This range includes sounds like those present in a 'quiet room' as well as very loud sounds which can cause ear damage.

Corrections for frontal or diffuse sound fields are made automatically by the analyzer according to the settings of front panel buttons. An overload light comes on when any of the analyzer's circuits are overdriven.

How to measure short, impulsive sounds — like the sound of a stamping machine or a single typewriter stroke — has always been one of the most vexing prob-

* The subscript G indicates loudness measured by the Zwicker method. Sones and phons without subscripts refer only to subjective tests using human observers.

lems in loudness measurement. Previously, the only way to analyze a single-shot phenomenon was to capture it on magnetic tape, make a tape loop and try to analyze the sound by playing it back over and over.

With the new loudness analyzer, impulsive sounds are no longer a serious problem. The analyzer has electronic storage circuits which can be called upon to 'remember' the peak loudness of the sounds occurring during any desired interval. If, as is usually the case, the single-shot sound is much louder than the background noise in the area, the loudness analysis stored by the analyzer will be that of the short sound.

Fig. 2 shows a typical analysis of a single typewriter stroke. The waveform of Fig. 2(a) is the sound pressure

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Fig. 3. *Loudness analyzer microphone consists of 1" condenser microphone cartridge and preamplifier. Response is flat within ±1 dB from 30 Hz to 8 kHz and within ±2 dB from 8 kHz to 18 kHz.*

as a function of time, photographed on a storage oscilloscope. The sound of the typewriter key hitting the paper is the large two-millisecond transient just to the left of the center of the photograph. Low frequencies, which make up most of the waveform, represent the sound of the typewriter carriage.

Fig. 2(b) is the Zwicker analysis of this typewriter stroke, photographed on the CRT of the new loudness analyzer. The contributions of the low and high frequency components to the total loudness are clearly visible. Total loudness, the area under the Zwicker diagram, was given by the analyzer's meter as 100 sones$_G$.

While the CRT display of Fig. 2(b) was being photographed, the analyzer was making the Zwicker diagram of Fig. 2(c) on an X-Y recorder. Sound pressure levels in each frequency band can be read from this diagram, along with loudness densities, loudness, loudness level, and so on.

Analyzing the loudness of this typewriter stroke and obtaining the CRT photograph and the recorder plot took less than two minutes.

Inputs

Sounds can be fed into the analyzer either by way of a microphone or directly from a tape recorder. The analyzer has its own microphone (Fig. 3), consisting of a 1" condenser microphone cartridge and a preamplifier. The microphone has a frequency response which is flat within ±1 dB from 30 Hz to 8 kHz and within ±2 dB from 8 kHz to 18 kHz. Its nominal sensitivity is 5 mV/μbar.

If desired, the microphone cartridge can be disconnected from the preamplifier and replaced with any standard 1" condenser microphone cartridge or with an adapter for some other input device. The analyzer has a built-in 200 V supply for condenser microphones and a built-in correction adjustment for microphones having sensitivities from 1 dB above 5 mV/μbar to 4.5 dB below mV/μbar.

How It Works

Fig. 4 is a block diagram of the loudness analyzer.

Input signals to the analyzer go first through an internal preamplifier. Then, if the sound field being measured is diffuse rather than frontal, the signals are weighted by a network which simulates the difference between the ear's sensitivity in a diffuse field and its sensitivity in a frontal field. This difference is standardized in ISO Recommendation 454.

Among its input circuits the analyzer has a built-in wideband noise generator (a neon lamp) which provides a check on the operation of the instrument. Fig. 5 shows what the display typically looks like when the front-panel CHECK button is pressed, provided that the instrument is working properly. At the same time, the deflection of the loudness meter should be in a section of its scale marked in blue. A display like Fig. 5 and a meter deflection in the blue region indicate that the instrument is operating within its overall accuracy specifications.

Filters Produce 20 Channels

Having gone through the input circuits, the electrical signals representing the sound to be analyzed are separated into 20 channels by active bandpass filters. The

Loudness Analyzer Aids Noise Reduction,
Production Testing, Speech Analysis

Zwicker's method of measuring subjective loudness (ISO R 532) is the most accurate of the modern methods, but it is so complicated that it used to take thirty minutes or more to make a Zwicker analysis of a sound. Now there is a faster way. The new loudness analyzer described in the accompanying article gives a Zwicker-type CRT display in less than a second and records a Zwicker diagram on paper in about 90 seconds.

Noise reduction is often greatly simplified if the Zwicker diagram for the noise can be obtained. The diagram is a plot of loudness density versus subjective pitch. It shows how sound components in each of 20 frequency bands contribute to the total loudness of a sound. Consequently, it shows how noise-reduction techniques can be applied most effectively. For example, low-frequency sounds are typically much harder to attenuate than high-frequency sounds. If the Zwicker diagram shows strong high-frequency components, the loudness of the noise can be reduced by means of relatively thin sound-absorbing material. One result can be lower costs.

Occasionally, the spectral analysis produced by the loudness analyzer may show that the most obvious sound-producing element in a mechanism contributes only slightly to loudness, but that some minor and easily altered element is a powerful offender. A resonant panel, easily damped, often turns out to be the worst offender, but without a spectral analysis, it might be nearly impossible to discover this.

Another requirement for effective noise abatement is the ability to make 'before' and 'after' comparisons to show how much or how little a particular technique has reduced the noise. The speed of the analyzer makes this a simple matter. Formerly, analysis often took most of the time.

Representative of the noise-reduction problems facing our noisy world are those encountered by

- Automobile manufacturers, who want to improve overall quietness in their cars or cut the cost of sound-proofing materials.
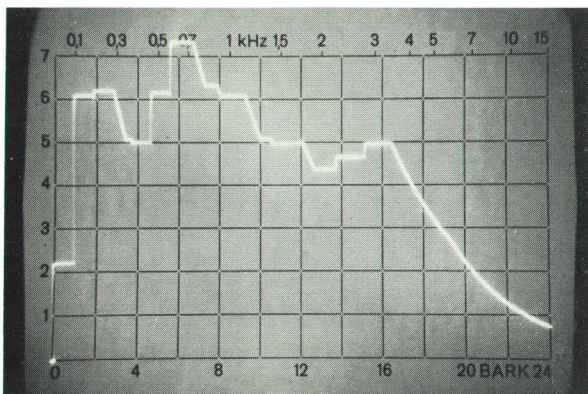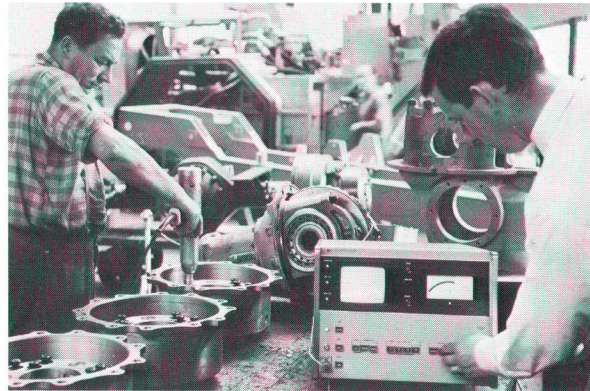
Analyzing factory noise.

- Office equipment manufacturers, who want to reduce the noisiness of their products. Typewriters, calculating machines, printers, etc., all have a characteristic hammer noise which is broadband and highly impulsive, and therefore difficult to evaluate by conventional means.

- The aircraft industry, which wants to improve passenger comfort and reduce airport noise.

- Broadcasters, who have trouble adjusting for differences in loudness between programs and commercials.

- Factory owners and manufacturers of heavy equipment, who want to avoid ear damage and increase productivity by cutting down on heavy machine noise. Many of these noises are of the impulsive variety (e.g., stamping machines).

- Railroads, which, like the aircraft industry, want to improve passenger comfort and reduce external noise, especially from switchyards.

- Architects, who want to design quieter apartments.

- Manufacturers of air conditioners and household appliances, such as electric razors and vacuum cleaners.

- Construction firms, which want to reduce the annoyance that pneumatic drills or riveters cause people nearby.

In speech analysis, loudness analysis finds application in studies of bandwidth compression, deafness, and speech intelligibility. An interesting technique that has been proposed for some of these studies is the possibility of making three-dimensional time/frequency/loudness diagrams by suitably processing and recording the outputs of the loudness analyzer.

In production testing or quality assurance the loudness analyzer brings an element of objectivity to tests which were formerly conducted by human listeners. Components which are tested this way are such things as gears and electric razors.

Loudness analysis of jet aircraft noise. Loudness = 400 sones$_G$
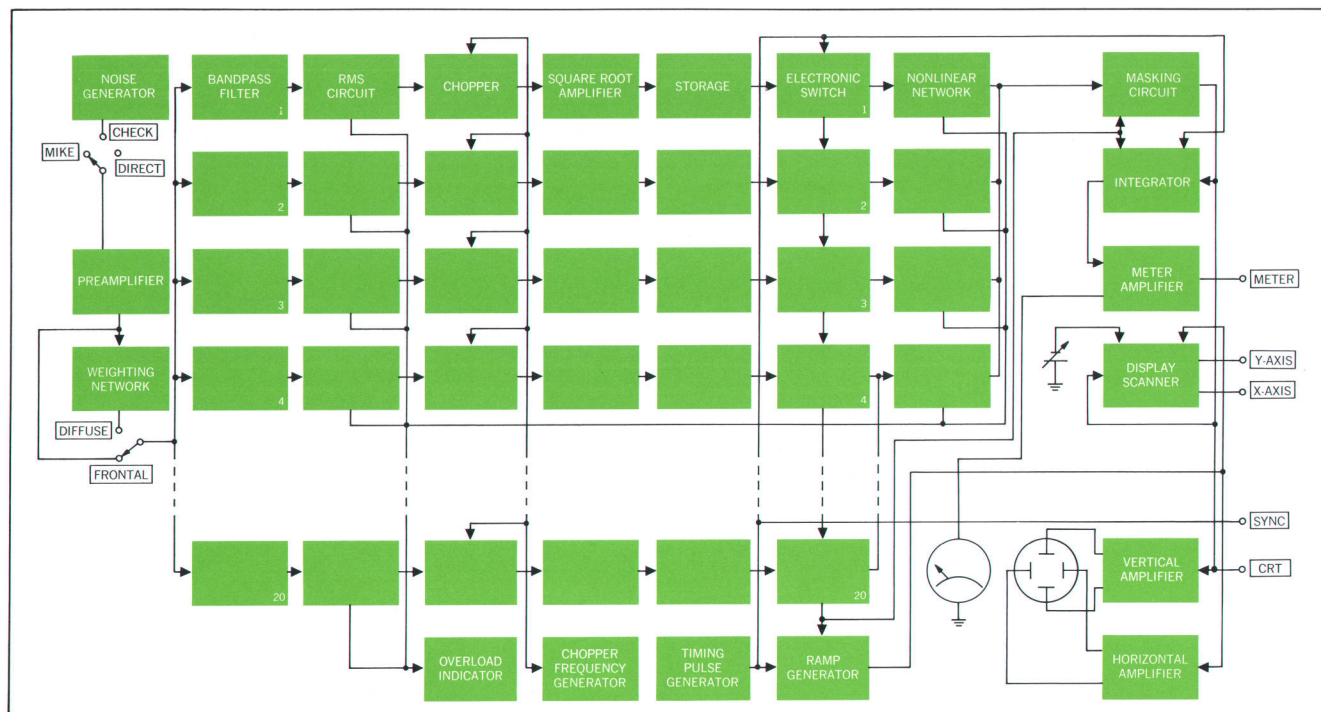
Fig. 4. *Model 8051A Loudness Analyzer splits frequency range from 45 Hz to 14 kHz into 20 bands which approximate the ear's critical bands. Signals in each band are scanned sequentially after processing to reproduce Zwicker diagrams on CRT or X-Y recorder.*

filters cover the frequency range between 45 Hz and 14 kHz as required by ISO Recommendation 532, Method B.

The channel filters are active third-order filters. Those in channels 1 and 2 are octave-bandwidth filters, and the channel 3 filter is a ⅔-octave filter. Each of these three filters consists of a bandpass section, a low-pass section, and a high-pass section, and has symmetrical roll-off characteristics above and below its passband.

The filters in channels 4 through 20 are unsymmetrical ⅓-octave filters, each consisting of one high-pass section and two low-pass sections. Fig. 6 is a typical response curve for one of the ⅓-octave filters. (A ⅓-octave filter has an upper 3 dB frequency which is $\sqrt[3]{2}$ times its lower 3 dB frequency.)

Also shown in Fig. 6 are the tolerances spelled out in international standards for ⅓-octave filter responses. The roll-off characteristics of the analyzer's filters exceed these standards by wide margins, thereby reducing the crosstalk between channels. Above the passband, for example, at the center frequency of the next higher filter, the attenuation of the analyzer filter is about 24 dB instead of the required 13 dB. Below the passband, the rolloff is not so steep (although it still far exceeds the standards). The filter was designed to be unsymmetrical because

crosstalk on the low side tends to be cancelled by the circuits which account for the ear's masking effect; hence the rolloff on the low side need not be so steep.

As the preceding article explains, the ⅓-octave bands correspond closely to the experimentally determined critical bands of the human ear. Sounds having bandwidths narrower than a critical band contribute to loudness in the same way that pure tones do. For this reason a finer analysis than a ⅓-octave analysis would not give any more accuracy in loudness measurements.

RMS Circuits

Following each filter is an rms circuit, which produces a dc signal proportional to the rms value of the filter output.

The rms circuit has a dynamic range of 60 dB and an integration time of about 100 ms. This integration time is approximately the time constant of the human ear. Thus the analyzer's response to sounds of short duration is similar to the ear's. The large dynamic range is necessary for two reasons. First, the analyzer's display range is 40 dB, so the dynamic range of the rms circuit has to be at least this large. Second, to give accurate results for impulsive sounds, the rms circuit must be able to handle

sounds with high crest factors (ratio of peak to rms). The circuit used in the analyzer will handle crest factors as high as seven.

From the rms circuit, signals go through a chopper, and then to a 'square-root' amplifier. The chopper allows the amplifier to be ac-coupled, thereby avoiding drift problems. It works at 15 kHz, fast enough to follow changes in the rms signal.

Band Loudness Density

The purpose of the square-root amplifier is to begin to convert the rms value of the sound pressure in the channel to something which can be used to compute loudness.

As the preceding article explains, a pure tone or a narrow-band sound excites nerves in the ear which correspond to a wide range of frequencies. Zwicker's approximation to this effect is a series of masking curves, like the one shown in Fig. 7, which is for a 1 kHz tone. A masking curve is a plot of loudness density versus subjective pitch, and the area under it represents the total loudness of the sound. When the masking curves of two sounds overlap, the total area under them is less than the sum of their individual areas, so the total loudness is less than the sum of the individual loudnesses.

Masking curves have two parts, a horizontal line in the frequency band where the sound is located, and a 'tail' which decays to zero with increasing pitch (frequency). We will call the height of the horizontal portion of the masking curve the *band loudness density*.

Band loudness density is a function of the rms sound pressure and the center frequency of the channel. The shape of this function is determined by the response of the human ear, which is usually expressed in the form of equal-loudness-level contours, as explained in the preceding article. Band loudness density can be expressed
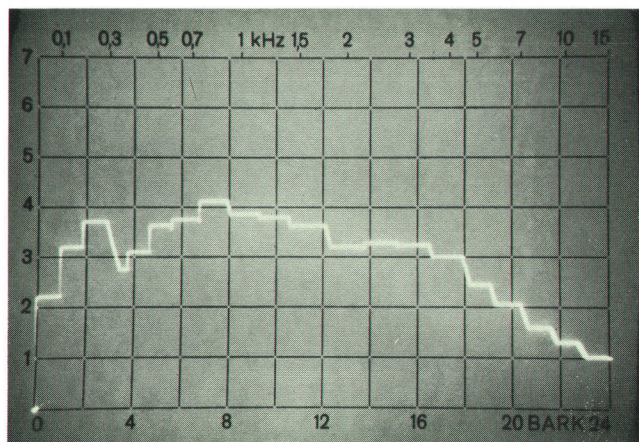

Fig. 5. *Typical loudness analyzer display in CHECK mode. In this mode, analyzer input comes from built-in wideband noise generator.*

as the product of two functions, one a function of frequency only, and the other a function of pressure and frequency. The former function is simulated in the loudness analyzer simply by varying the gain of the rms circuits from channel to channel. The latter function is illustrated in Fig. 8.

Fig. 8 shows *normalized* band loudness density as a function of rms sound pressure and center frequency. 'Normalized' means that loudness variations which are functions of frequency only are not included in Fig. 8. The normalization is with respect to the ear's response at 1 kHz. For 1 kHz, but not for other center frequencies, the height of the horizontal portion of the masking curve can be determined from Fig. 8. Taking the example of Fig. 7, we have a 1 kHz tone which has a loudness of 37 sones$_G$, corresponding to a loudness level of 92 phons$_G$ and a sound pressure level of 92 dB. According to Fig. 8 such a sound has a band loudness density of about 9.2 sones$_G$ per Bark. This is the height of the horizontal portion of the masking curve, Fig. 7.*

The square-root amplifier converts rms sound pressure to a first approximation to normalized band loudness density. At high sound pressures and at high center frequencies, all of the curves of Fig. 8 approach the dashed curve, which is a straight line with a slope of approximately 0.5. Translated mathematically, this means that normalized band loudness density is approximately pro-

portional to the square root of the rms sound pressure in the channel. Actually, the slope is 0.47, so the amplifier's response is not quite a square-root function.

Deviations from the dashed line at low frequencies and at low sound pressure levels are taken care of in the loudness analyzer by additional nonlinear networks, which we will say more about later.

Analog Storage

After being shaped in the 'square-root' amplifier, the signal in each channel goes to a storage circuit, where it may or may not be stored. If the INSTANT button on the front panel is depressed, the signal is not stored. If the PEAK button is depressed, the circuit will retain the highest value of the signal that occurs while the button is depressed. This means that, as long as the PEAK button is depressed, the stored voltage will increase whenever the square-root amplifier output exceeds the voltage already in the storage circuit.
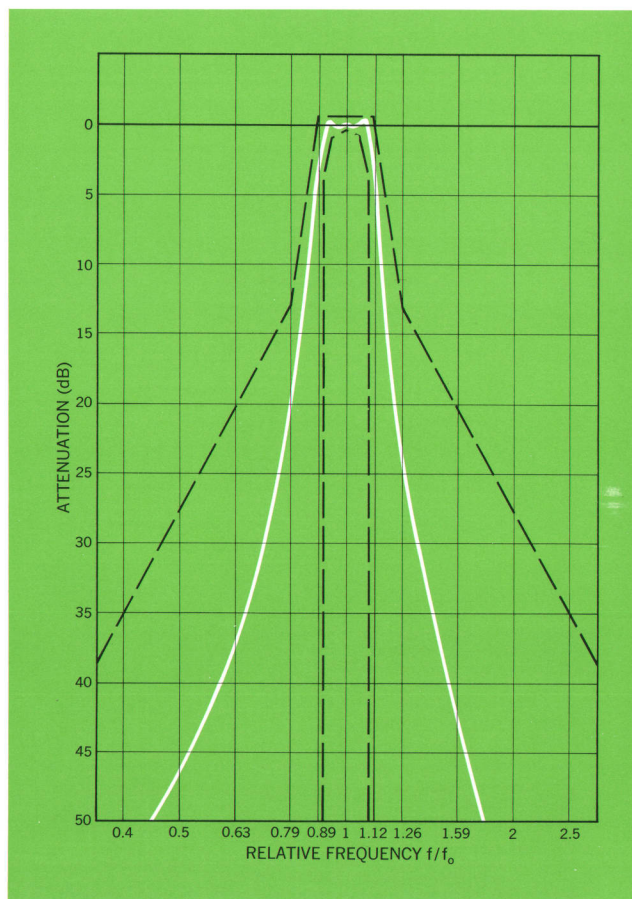

Fig. 6. *Typical response curve for ⅓-octave filter used in loudness analyzer. Response is down more than 20 dB at center frequency of following filter, and down 60 dB at twice the center frequency. Dashed curves show tolerances allowed by international standards.*
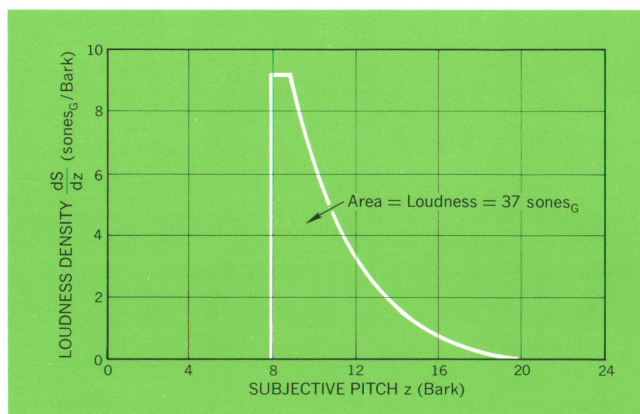
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* However, if you look on the Zwicker diagram, Fig. 2(c), you will not find a band loudness density of 9.2 sones$_G$ per Bark for a sound pressure level of 92 dB and a center frequency of 1 kHz. Instead, you will find about 8 sones$_{GF}$ per Bark (frontal sound field). This is because, in its ISO-recommended form, Zwicker's procedure has been modified to use ⅓-octave filters instead of critical-band filters. The ⅓-octave filter centered at 1 kHz has a bandwidth of 220 Hz instead of the critical bandwidth of 160 Hz. The diagram accounts for this difference.

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Fig. 7. *Masking curve for 1 kHz tone. This is Zwicker's approximation to the way in which a pure tone or narrowband sound (centered at 1 kHz, in this case) excites nerves in the ear corresponding to many frequencies. Masking occurs if the curves corresponding to two or more sounds overlap. Loudness analyzer simulates masking effect by means of a capacitor-discharge circuit which follows tail of curve.*

If the HOLD button on the front panel is depressed, the storage circuit will retain whatever the square-root amplifier output happens to be at the time the button is pressed.

The storage circuits are analog circuits, using capacitors as the storage elements. They have time constants of the order of one hour, long enough to insure that the stored signals will not change by more than 5% of full scale in two minutes.

The HOLD and PEAK functions are remotely programmable.

Read Out

Information in the storage circuits is read out 40 times per second, beginning with channel 1 and going through channel 20. The CRT sweep is synchronized with the read sequence.

Each storage circuit is followed by an electronic switch which is driven by a one-shot multivibrator. A timing-pulse generator triggers the channel 1 multivibrator every 25 ms. The channel-1 multivibrator triggers the channel-2 multivibrator, and so on. When the multivibrator corresponding to a given channel is triggered, the storage circuit in that channel is connected to the output circuits via its electronic switch. Only one storage circuit at a time is connected to the output circuits.

The horizontal scale of the Zwicker diagram is linear in subjective pitch, which is a very nonlinear function of the logarithm of frequency. Therefore, although most of the channels are ⅓-octave wide, each channel has a different width on the Zwicker diagram. To give all channels the proper widths on the display, the multivibrators which drive the electronic switches all have different periods.

Masking Effect Simulated

The 20 outputs of the electronic switches are shaped further in four nonlinear networks. These networks finish the conversion of rms sound pressure to band loudness density, i.e. the height of the horizontal portion of Zwicker's masking curve (Fig. 7). Recall that the 'square-root' amplifiers start the conversion, but leave undone some necessary shaping for low-level and low-frequency signals.

From the nonlinear networks, signals are sent to a masking circuit consisting of a peak detector, a capacitor-discharge network, and a buffer amplifier. This circuit accounts for the masking of low-level high-frequency sounds by high-level low-frequency sounds. It does this by producing an output which follows the 'tails' of Zwicker's masking curves, Fig. 7.

Heinz Blässer

Heinz Blässer graduated from the Technical University of Berlin with the degree of Diplom-Ingenieur in physics in 1963. Before joining HP in 1965, he worked as a patent engineer. Heinz started the 8051A Loudness Analyzer project in mid-1965, and he is now in charge of the acoustical instrumentation group.

Heinz is a member of the German Physical Society.
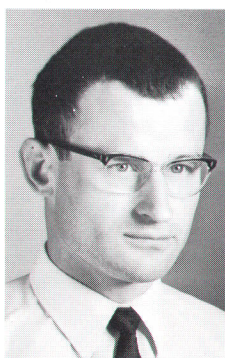
Helmut Finckh

Helmut Finckh graduated from the Technical University of Stuttgart with the degree of Diplom-Ingenieur in electrical engineering in 1965. After joining HP in 1965, he worked as a development engineer, responsible for the design of active filters. Since 1966 he has been the project leader for the 8051A Loudness Analyzer.
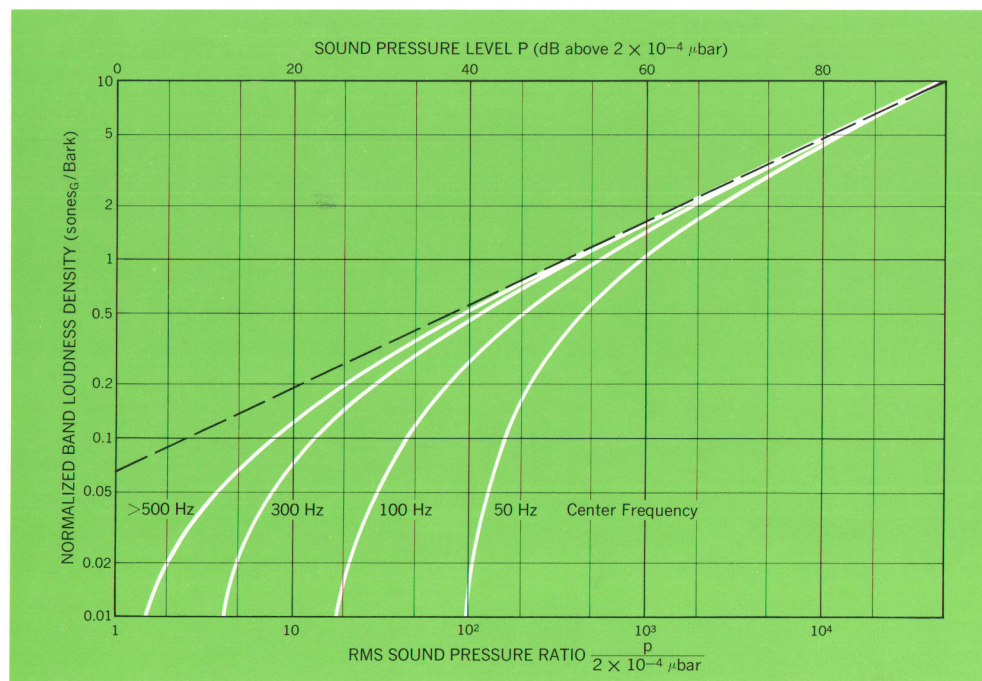
Fig. 8. *Height of horizontal portion of Zwicker's curve (see Fig. 7), called band loudness density, is a function of rms sound pressure in channel and center frequency of channel. This function can be expressed as the product of two functions. One is a function of frequency only. The other is shown here as 'normalized' band loudness density. Dashed curve has slope of 0.47; this means that at high sound pressure levels, normalized band loudness density is approximately proportional to square root of rms sound pressure.*

At the start of each sweep the masking-circuit capacitor is charged by the peak detector to a value proportional to the band loudness density of this channel. If the band loudness density in the next channel is higher, then when the next channel is sampled, the capacitor is charged to a voltage proportional to the higher band loudness density. If it is lower, then the capacitor is discharged until a band loudness density in one of the next channels is higher than the loudness density corresponding to the voltage remaining on the capacitor. The time constant of the discharge varies with the frequency band and the loudness density, and may change from one value to another during any particular discharge.

CRT, Meter, and Recorder Outputs

The output of the masking circuit is displayed on the CRT. It is also supplied to a display scanner which has X and Y recorder outputs. Pressing the HOLD button and starting the display scanner will produce an accurate Zwicker plot on an X-Y recorder in about 90 seconds.

Besides the CRT display, which shows the contribution of each frequency band to loudness, the analyzer has a meter output which displays total loudness. The meter indicates the output of an integrator which computes the integral of the CRT display in sones$_G$. Another recorder output supplies a voltage proportional to the meter reading, for strip-chart recording.

Acknowledgments

1968 UTC Offset Announced

The International Bureau of Time, Paris, has announced that the fractional frequency offset for Coordinated Universal Time for 1968 will continue to be −300 parts in 10^{10}. This offset from the atomic time (and frequency) scale is annually selected so that Coordinated Universal Time (UTC) can approximate UT2, a time scale related to the rotation of the earth.

For the past two years the offset has also been −300 parts in 10^{10}. Before that it was −150 parts in 10^{10} and, earlier still, −130 parts in 10^{10}.

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jmiles@pop.net